Statistics Knowledge Example: 1-way ANOVA

I randomly selected 6 students from 4 different 320 classes in the psychology department taught by 4 different instructors and gave them the same stat test to see how well my students are doing compared to other professor's students. On this particular stat test student scores' in general are normally distributed. Are the students' scores different across the 4 professors?

	Prof X	Prof Y	Prof Z	Me	
	69	56	67	73	
	74	81	88	85	
	56	71	78	82	
	63	63	89	77	
	65	79	86	90	
	55	73	69	100	
Mean	**63.667**	**70.500**	**79.500**	**84.500**	**GM = 74.542**
Sum	**382**	**423**	**477**	**507**	**Total=1789**
SD	**7.367**	**9.545**	**9.731**	**9.649**	**ΣY^2 = 136571**

I. Test for significance using all 7 significance testing steps.

1. **State Null Hypothesis:**

$$h_0 : \mu_{\text{Professor X}} \_\_\_ \mu_{\text{Professor Y}} \_\_\_ \mu_{\text{Professor Z}} \_\_\_ \mu_{\text{My Students}}$$

2. **Alternative Hypothesis:** $h_1 : At\ least\ 2\ \ \mu s\ are\ different$

3. **Decide on α (usually .05):** α = ____

4. **Decide on type of test (distribution; *z*, *t*, F, etc.)**
   Questions to ask:
   a. How many groups do you have?
      i. Only 2 than you can use a t-test.
      ii. More than 2 groups → ANOVA → F distribution
   b. Can we treat the scores as independent (e.g. they are NOT from the same person, they are NOT matched subjects, they are NOT related subjects, etc.)?
      If Yes, then continue with the between groups ANOVA
      If No, STOP you may need to perform a repeated measures ANOVA
   c. Can we assume a normally distributed sampling distribution?
      In other words, do we have 20+ degrees of freedom for the WG source of variance?
      If yes, then continue.
      If no, do not continue the test cannot be performed.
   d. Do the groups have homogenous variances?

$$F_{MAX} = \frac{s^2_{\text{Largest}}}{s^2_{\text{Smallest}}} = \frac{\_\_\_}{\_\_\_} = \_\_\_\_\_$$, if this value is smaller than 3 proceed.

5. **Find critical value & state decision rule**
   a. For F_{cv} you need both df_{BG} = #groups – 1 and df_{WG} = *N - #groups*. Table D.3 F_{cv} (df_{BG}, df_{WG}), if F_o > F_{cv} reject the null hypothesis
   b. So df_{BG} = ____ – 1 = ____ and df_{WG} = ____ - 1 = 20. Table D.3 F_{cv} (___,___) = ______, if F_o > _______ reject the null hypothesis
6. **Calculate test**
   a. You can use the Deviation Approach

$SS_{BG} = \sum n_j \left(\bar{Y}_j - \bar{Y}_{GM}\right)^2 = [\_\_\_ * (\_\_\_\_\_ - \_\_\_\_\_)^2] + [\_\_\_ * (\_\_\_\_\_ - \_\_\_\_\_)^2] +$

$+[\_\_\_\_ * (\_\_\_\_\_ - \_\_\_\_\_)^2] + [\_\_\_ * (\_\_\_\_\_ - \_\_\_\_\_)^2] =$

$SS_{BG} = \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_ = \_\_\_\_\_\_$

$SS_{WG} = \sum \left(Y_i - \bar{Y}_j\right)^2 = (\_\_\_\_\_ - 63.667)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 +$

$(\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 +$

$(\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 +$

$(\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 +$

$(\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 =$

$SS_{WG} = \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ +$

$+ \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ +$

$+ \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_$

$+ \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ +$

$+ \_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_\_\_$

$SS_{Total} = \sum \left(Y_i - \bar{Y}_{GM}\right)^2 = (\_\_\_ - 74.542)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 +$

$(\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 +$

$(\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 +$

$(\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 +$

$(\_\_\_ - \_\_\_\_\_)^2 + (\_\_\_ - \_\_\_\_\_)^2 =$

$SS_{Total} = \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ +$

$+ \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ +$

$+ \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_$

$+ \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ +$

$+ \_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_\_\_$

b. Or you can use the computational formulas

$$SS_{BG} = \frac{\sum\left(\sum a_j\right)^2}{n} - \frac{T^2}{N} = \frac{382^2 + \_\_\_\_^2 + \_\_\_\_^2 + \_\_\_\_^2}{\_\_\_} - \frac{\_\_\_\_^2}{\_\_\_} =$$

$$SS_{BG} = \frac{\_\_\_\_\_\_}{\_\_\_} - \frac{\_\_\_\_\_\_}{\_\_\_} = \_\_\_\_\_\_ - \_\_\_\_\_\_ = \_\_\_\_\_\_$$

$$SS_{WG} = \sum Y^2 - \frac{\sum\left(\sum a_j\right)^2}{n} = 136571 - \_\_\_\_\_\_ = \_\_\_\_\_\_$$

$$SS_T = \sum Y^2 - \frac{T^2}{N} = 136571 - \_\_\_\_\_\_ = \_\_\_\_\_\_$$

c. Putting the SS and DF into an ANOVA summary table we can calculate the F-observed value.

Source	SS	df	MS	F
BF	________	__	________	6.204
WG	________	___	________	
Total	________	___		

7. **Apply Decision Rule**
Since, ________ (i.e. observed value) _____ (i.e. >, <) ________ (critical value), __________(i.e. **DO or DO NOT)** reject the null hypothesis.

II. Compare Professor X's students to my students using the Tukey critical difference (CD).

1. **State Null Hypothesis** $h_0 : \mu_{\text{Professor X}} = \mu_{\text{My Students}}$

2. **Alternative Hypothesis** $h_1 : \mu_{\text{Professor X}} \neq \mu_{\text{My Students}}$

3. **Decide on α (usually .05)** α = ____

4. **Find critical value & state decision rule**

$$CD = q\sqrt{\frac{MS_{WG}}{n}} = \_\_\_\_\_\sqrt{\frac{\_\_\_\_\_}{\_\_\_}} = \_\_\_\_\_$$

(Note: Q is found in a q table and you need alpha, the number of groups and the WG degrees of freedom.)

$$If\ \ |MeanDiff| > CD,\ reject\ h_0.$$

$$If\ \ \left|\bar{X}_{\Pr ofX} - \bar{X}_{MyStudents}\right| > \_\_\_\_\_,\ reject\ h_0.$$

5. **Calculate test**

$$\left|\bar{X}_{\text{Prof X}} - \bar{X}_{MyStudents}\right| = \left|\_\_\_\_\_\_ - \_\_\_\_\_\_\right| = \_\_\_\_\_\_$$

6. **Apply decision rule**

Since, _______ (i.e. observed Mean Difference) ____ (i.e. >, <) _______ (CD value), _________(i.e. **DO or DO NOT)** reject the null hypothesis.

III. Calculate both η^2 and ω^2

1. $$\eta^2 = \frac{SS_{BG}}{SS_{Total}} = \frac{\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_} = \_\_\_\_\_\_\_$$

2. $$\omega^2 = \frac{SS_{BG} - (k-1)MS_{WG}}{SS_{Total} + MS_{WG}} = \frac{\_\_\_\_\_\_ - [(\_\_\_ - 1) * \_\_\_\_\_\_]}{\_\_\_\_\_\_ + \_\_\_\_\_\_} = \_\_\_\_\_$$